No. of Printed Pages: 6

CLASS: XII
POST MID-TERM EXAM (2025-26)
SUBJECT: MATHEMATICS (041)
SET-B2

Time: 3 Hrs. **Max. Marks: 80**

General Instructions:

1. This Question paper contains five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 case-based assessment (4 marks) with sub parts.
7. There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E

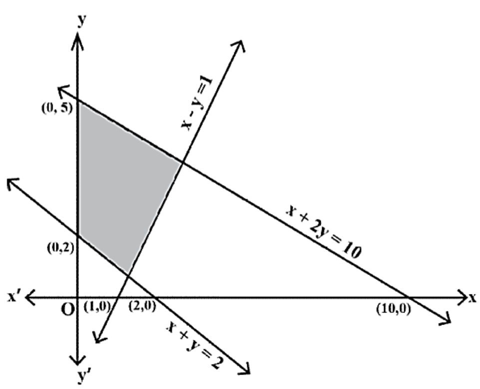
Section-A

(Multiple Choice Questions) Each question carries 1 mark

1. If A and B are independent events, then $P(\overline{A}/\overline{B}) =$

(a) $P(A/\overline{B})$ (b) $1 - P(A)$ (c) $1 - P(B)$ (d) $1 - P(\overline{A}/B)$

2. The feasible region corresponding to the linear constraints of a Linear Programming Problem is given below.



Which of the following is not a constraint to the given Linear Programming Problem?

(a) $x + y \geq 2$ (b) $x + 2y \leq 10$ (c) $x - y \geq 1$ (d) $x - y \leq 1$

3. The objective function $Z = ax + by$ of an LPP has maximum value 36 at $(4, 4)$ and minimum value 15 at $(3, 1)$. Which of the following is true?

(a) $a = 3, b = 5$ (b) $a = 3, b = 6$ (c) $a = 2, b = 7$ (d) $a = 5, b = 3$

4. The value of $\hat{\imath}\,.\left(\hat{k}\times\hat{\jmath}\right)+\hat{\jmath}\,.\left(\hat{\imath}\times\hat{k}\right)+\hat{k}.\left(\hat{\imath}\times\hat{\jmath}\right)$

(a) -3 (b) -2 (c) -1 (d) 0

5. If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a}|=1, |\vec{b}|=2$ and $\vec{a}\,.\vec{b}=\sqrt{3}$, then the angle between $2\vec{a}$ and $-\vec{b}$ is:

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{5\pi}{6}$ (d) $\frac{11\pi}{6}$

6. If vector $\vec{a}=3\,\hat{\imath}+2\,\hat{\jmath}-\hat{k}$ and vector $\vec{b}=\hat{\imath}-\hat{\jmath}+\hat{k}$ then which of the following is correct?

(a) $\vec{a}\parallel\vec{b}$ (b) $\vec{a}\perp\vec{b}$ (c) $|\vec{b}|>|\vec{a}|$ (d) $|\vec{a}|=|\vec{b}|$

7. $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} sin^2x\ dx$ is equal to:

(a) $\frac{\pi}{2}$ (b) 0 (c) $\frac{\pi}{4}-2$ (d) $\frac{\pi}{4}-\frac{1}{2}$

8. Given the integral $\int x^4\, sin^6(x^5)\cos(x^5)\,dx = a\, sin^7(x^5)+\mathrm{C}$, the value of a is

(a) $\frac{1}{15}$ (b) $\frac{1}{20}$ (c) $\frac{1}{30}$ (d) $\frac{1}{35}$

9. The integrating factor of the differential equation $(3x^2+y)\frac{dx}{dy}=x$ is:

(a) $\frac{1}{x}$ (b) $\frac{1}{x^2}$ (c) $\frac{2}{x}$ (d) $-\frac{1}{x}$

10. The value of 'λ' so that $f(x)=\sin x-\cos x-\lambda x$ decreases for all real values of x are:

(a) $1<\lambda<\sqrt{2}$ (b) $\lambda\geq 1$ (c) $\lambda\geq\sqrt{2}$ (d) $\lambda<1$

11. If $y=x^2|x|$, then $\frac{dy}{dx}$ at $x=-4$ is

(a) not defined (b) can't be calculated (c) -48 (d) 48

12. The number of points, where $f(x)=[x], 0<x<3$ ([.] Denotes greatest integer function) is not differentiable is:

(a) 1 (b) 2 (c) 3 (d) 4

13. If $\begin{bmatrix}1 & 2 & 1\\ 2 & 3 & 1\\ 3 & a & 1\end{bmatrix}$ is non-singular matrix and $a\in A$, then the set A is:

(a) $\boldsymbol{R}$ (b) $\{0\}$ (c) $\{4\}$ (d) $\boldsymbol{R}-\{4\}$

14. For any square matrix A of order 3, if $A.\mathrm{adj}(A)=\begin{bmatrix}8 & 0 & 0\\ 0 & 8 & 0\\ 0 & 0 & 8\end{bmatrix}$, then the value of $|A|$ is

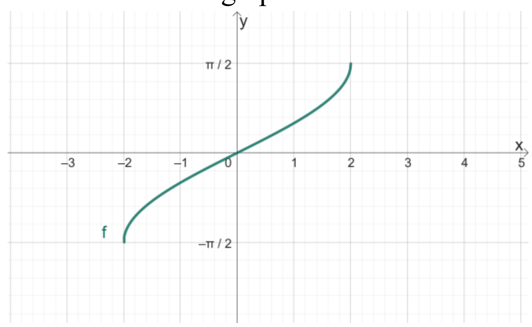
(a) 4 (b) 16 (c) 64 (d) 8

15. If A is a square matrix of order 2 such that $|A| = 4$, then $|4 \text{ adj } A|$ is equal to:
(a) 16 (b) 64 (c) 256 (d) 512

16. If $A = \begin{bmatrix} 7 & 0 & x \\ 0 & 7 & 0 \\ 0 & 0 & y \end{bmatrix}$ is a scalar matrix, then y^x is equal to
(a) 0 (b) 1 (c) 7 (d) ± 7

17. If matrix $A = [a_{ij}]_{2\times 2}$, where $a_{ij} = 1$ if $i \neq j$ and 0 if $i = j$ then A^2 is equal to
(a) I (b) A (c) O (d) $-A$

18. Identify the function shown in the graph:



(a) $\sin^{-1} x$ (b) $\sin^{-1} 2x$ (c) $\sin^{-1} \frac{x}{2}$ (d) $2\sin^{-1} x$

Assertion-Reasoning Based Question:
In the questions 19, 20 a statement of **Assertion (A)** is followed by a statement of **Reason (R).** Choose the correct answer out of the following choices.
(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true and R is not the correct explanation of A.
(c) A is true but R is false. (d) A is false but R is true.

19. **Assertion (A):** If $|\vec{a} \times \vec{b}|^2 + |\vec{a} . \vec{b}|^2 = 256$ and $|\vec{b}| = 8$, then $|\vec{a}| = 2$.
Reason (R): $\sin^2\theta + \cos^2\theta = 1$ and $|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin\theta$ and $|\vec{a} . \vec{b}| = |\vec{a}||\vec{b}| \cos\theta$

20. **Assertion (A):** The value of $\cos\left[\frac{\pi}{2} + \sin^{-1}\left(-\frac{1}{2}\right)\right] = \frac{1}{2}$.
Reason (R): $\sin^{-1}(-x) = \sin^{-1}(x)$

<u>Section-B</u>
(This section comprises 5 very short answer type-questions (VSA) of 2 marks each.)

21. Express the vector $\vec{a} = 5\hat{\imath} - 2\hat{\jmath} + 5\hat{k}$ as the sum of two vectors such that one is parallel to the vector $\vec{b} = 3\hat{\imath} + \hat{k}$ and the other is perpendicular to $\vec{b}$.

22. If $f(x) = \begin{cases} 2x-3, & -3 \le x \le -2 \\ x+1, & -2 < x \le 0 \end{cases}$, Check the differentiability of $f(x)$ at $x = -2$.

23. (a) Find $\int e^x(\cos x - \sin x)\operatorname{cosec}^2 x\, dx$

OR

23. (b) Sketch the graph of $y = |x-1|$ and evaluate $\int_0^2 |x-1|\, dx$.

24. If $xy = e^{x-y}$, then show that $\frac{dy}{dx} = \frac{y(x-1)}{x(y+1)}$

25. (a) If $\cot^{-1}(3x+5) > \frac{\pi}{4}$, then find the range of the values of x.

OR

25. (b) Find the value of $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left[\sin\left(-\frac{\pi}{2}\right)\right]$

<u>Section-C</u>

(This section comprises of short answer type-questions (SA) of 3 marks each.)

26. Consider the experiment of tossing a coin. If the coin shows head, toss it again; but if it shows a tail, then throw a die. Find the conditional probability of the event A: 'the die shows a number greater than 3' given that B: 'there is at least one tail'.

27. Solve the following LPP graphically:
Minimize: $Z = -3x + 4y$
Subject to the constraints: $x + 2y \le 8$
$3x + 2y \le 12$
$x \ge 0,\ y \ge 0$

28. (a) A line passing through the point A with position vector $\vec{a} = 4\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$ is parallel to the vector $\vec{b} = 2\hat{\imath} + 3\hat{\jmath} + 6\hat{k}$. Find the length of perpendicular drawn on this line from a point P with position vector $\vec{r_1} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$.

OR

28. (b) Find the vector and cartesian equation of the line passing through the point $(1, 2, -4)$ and perpendicular to two lines
$\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$

29. (a) Sketch the region bounded by the lines $2x + y = 8$, $y = 2$, $y = 4$ and the y-axis. Hence, obtain its area using integration.

OR

29. (b) Find the area of the following region using integration:
$\{(x, y): y^2 \le 2x \text{ and } y \ge x - 4\}$

30. A kite is flying at a height of 3 metres and 5 metres string is out. If the kite is moving away horizontally at the rate of 200 cm/s, find the rate at which string is being released.

31. (a) If $x = a\sin^3\theta,\ y = b\cos^3\theta$, then find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{4}$.

OR

31. (b) If $x^{13}y^7 = (x+y)^{20}$, then prove that $\frac{dy}{dx} = \frac{y}{x}$

Section-D

(This section comprises of long answer type-questions (LA) of 5 marks each.)

32. Find the distance between the line $\frac{x}{2} = \frac{2y-6}{4} = \frac{1-z}{-1}$ and another line parallel to it passing through the point $(4, 0, -5)$.

33. (a) If a curve $y = f(x)$, passing through the point $(1, 2)$ is the solution of differential Equation: $2x^2dy = (2xy + y^2)dx$, find the value of $f\left(\frac{1}{2}\right)$

OR

33. (b) If $\cos x\frac{dy}{dx} - y\sin x = 6x, 0 < x < \frac{\pi}{2}$ and $y\left(\frac{\pi}{3}\right) = 0$. Find the value of $y\left(\frac{\pi}{6}\right)$.

34. (a). Evaluate: $\int_0^{\frac{\pi}{4}} \frac{dx}{\cos^3 x\sqrt{2\sin 2x}}$

OR

34. (b) Find $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x + \cos x}{\sqrt{\sin 2x}}\, dx$

35. If $A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix}$, find A^{-1} and use it to solve the following system of Equations: $x - 2y = 10; 2x - y - z = 8;\ -2y + z = 7$

Section-E

(This section comprises of 3 case-based questions of 4 marks each with sub parts)

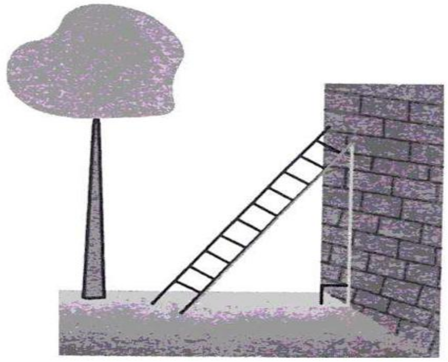
36. A shop selling electronic items sells smartphones of only three reputed companies A, B and C because chances of their manufacturing a defective smartphone are only 5%, 4% and 2% respectively. In his inventory he has 25% smartphones from company A, 35% smartphones from company B and 40% smartphones from company C.

A person buys a smartphone from this shop.

(i) Find the probability that it was defective.

(ii) What is the probability that this defective smartphone was manufactured by company B?

37. A ladder of fixed length 'h' is to be placed along the wall such that it is free to move along the height of the wall.



Based upon the above information, answer the following questions:

(i) Express the distance (y) between the wall and foot of the ladder in terms of 'h' and height (x) on the wall at a certain instant. Also, write an expression in terms of h and x for the area (A) of the right triangle, as seen from the side by an observer.

(ii) Find the derivative of the area (A) with respect to the height on the wall (x), and find its critical point.

(iii) (a) Show that the area (A) of the right triangle is maximum at the critical point.

OR

(iii) (b) If the foot of the ladder whose length is 5 m, is being pulled towards the wall such that the rate of decrease of distance (y) is 2 m/s, then at what rate is the height on the wall (x) increasing, when the foot of the ladder is 3 m away from the wall?

38. A class-room teacher is keen to assess the learning of her students the concept of "relations" taught to them. She writes the following five relations each defined on the set A = {1, 2, 3}:

R_1 = {(2, 3), (3, 2)}
R_2 = {(1, 2), (1, 3), (3, 2)}
R_3 = {(1, 2), (2, 1), (1, 1)}
R_4 = {(1, 1), (1, 2), (3, 3), (2, 2)}
R_5 = {(1, 1), (1, 2), (3, 3), (2, 2), (2, 1), (2, 3), (3, 2)}

The students are asked to answer the following questions about the above relations:

(i) Identify and justify the relation which is reflexive, transitive but not symmetric.

(ii) Identify and justify the relation which is reflexive and symmetric but not transitive.

(iii) (a) Identify and justify the relations which are symmetric but neither reflexive nor transitive.

OR

(iii) (b) What pairs should be added to the relation R_2 to make it an equivalence relation? Also find the number of equivalence relations possible on set A