No. of Printed Pages: 14

CLASS: XII
POST MID-TERM EXAM (2025-26)
SUBJECT: MATHEMATICS (041)
SET-B1/B2 MARKING SCHEME

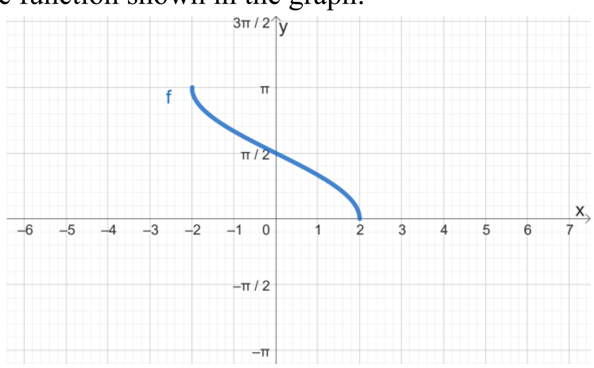
Time: 3 Hrs. **Max. Marks: 80**

General Instructions:

1. This Question paper contains **five sections** A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. **Section A** has **18 MCQ's** and **02** Assertion-Reason based questions of 1 mark each.
3. **Section B** has **5 Very Short Answer** (VSA)-type questions of 2 marks each.
4. **Section C** has **6 Short Answer (SA)**-type questions of 3 marks each.
5. **Section D** has **4 Long Answer (LA)**-type questions of 5 marks each.
6. **Section E** has 3 **case-based assessment** (4 marks) with sub parts.
7. There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E

Section-A
(Multiple Choice Questions) Each question carries 1 mark

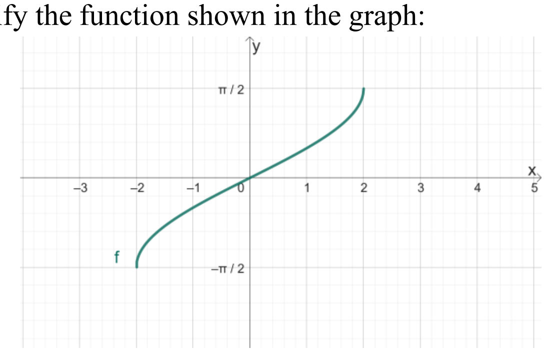
1. Identify the function shown in the graph:



(a) $\cos^{-1} x$ (b) $\cos^{-1} 2x$ (c) $\cos^{-1} \frac{x}{2}$ (d) $2\cos^{-1} x$

Sol. (c)

SET B2 /18. Identify the function shown in the graph:



(a) $\sin^{-1} x$ (b) $\sin^{-1} 2x$ (c) $\sin^{-1} \frac{x}{2}$ (d) $2\sin^{-1} x$

Sol. (c)

2/17. If matrix $A = [a_{ij}]_{2\times 2}$, where $a_{ij} = 1$ if $i \neq j$ and 0 if $i = j$ then A^2 is equal to

(a) I (b) A (c) O (d) $-A$

Sol. (a)

3/16. If $A = \begin{bmatrix} 7 & 0 & x \\ 0 & 7 & 0 \\ 0 & 0 & y \end{bmatrix}$ is a scalar matrix, then y^x is equal to

(a) 0 (b) 1 (c) 7 (d) ± 7

Sol. (b)

4/15. If A is a square matrix of order 2 such that $|A| = 4$, then $|4 \text{ adj } A|$ is equal to:

(a) 16 (b) 64 (c) 256 (d) 512

Sol. (b)

5/14. For any square matrix A of order 3, if $A.\text{adj}(A) = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix}$, then the value of $|A|$ is

(a) 4 (b) 16 (c) 64 (d) 8

Sol. (d)

6/13. If $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & a & 1 \end{bmatrix}$ is non-singular matrix and $a \in A$, then the set A is:

(a) $\boldsymbol{R}$ (b) $\{0\}$ (c) $\{4\}$ (d) $\boldsymbol{R} - \{4\}$

Sol. (d)

7. If $f(x) = \begin{cases} 3x - 2, & 0 < x \leq 1 \\ 2x^2 + ax, & 1 < x < 2 \end{cases}$ is continuous for $x \in (0, 2)$, then a is equal to:

(a) -4 (b) $-\frac{7}{2}$ (c) -2 (d) -1

Sol. (d)

SET B2 /12. The number of points, where $f(x) = [x], 0 < x < 3$ ([.] Denotes greatest integer function) is not differentiable is:

(a) 1 (b) 2 (c) 3 (d) 4

Sol. (b)

8. If $y = x|x|$, then $\frac{dy}{dx}$ at $x = -3$ is

(a) not defined (b) can't be calculated (c) 6 (d) -6

Sol. (c)

SET B2 /11 If $y = x^2|x|$, then $\frac{dy}{dx}$ at $x = -4$ is

(a) not defined (b) can't be calculated (c) -48 (d) 48

Sol. (c)

9/10. The value of 'λ' so that $f(x) = \sin x - \cos x - \lambda x$ decreases for all real values of x are:

(a) $1 < \lambda < \sqrt{2}$ (b) $\lambda \geq 1$ (c) $\lambda \geq \sqrt{2}$ (d) $\lambda < 1$

Sol. (c)

10/9. The integrating factor of the differential equation $(3x^2 + y)\frac{dx}{dy} = x$ is:

(a) $\frac{1}{x}$ (b) $\frac{1}{x^2}$ (c) $\frac{2}{x}$ (d) $-\frac{1}{x}$

Sol. (a)

11/8. Given the integral $\int x^4\, sin^6(x^5) \cos(x^5)\, dx = a\, sin^7(x^5) + C$, the value of a is

(a) $\frac{1}{15}$ (b) $\frac{1}{20}$ (c) $\frac{1}{30}$ (d) $\frac{1}{35}$

Sol. (d)

12/7. $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} sin^2 x\ \ dx$ is equal to:

(a) $\frac{\pi}{2}$ (b) 0 (c) $\frac{\pi}{4} - 2$ (d) $\frac{\pi}{4} - \frac{1}{2}$

Sol. (d)

13/6. If vector $\vec{a} = 3\,\hat{\imath} + 2\,\hat{\jmath} - \hat{k}$ and vector $\vec{b} = \hat{\imath} - \hat{\jmath} + \hat{k}$ then which of the following is correct?

(a) $\vec{a} \parallel \vec{b}$ (b) $\vec{a} \perp \vec{b}$ (c) $|\vec{b}| > |\vec{a}|$ (d) $|\vec{a}| = |\vec{b}|$

Sol. (b)

14. The unit vector(s) perpendicular to the vectors $\hat{\imath} - \hat{\jmath}$ and $\hat{\imath} + \hat{\jmath}$ is:

(a) $\hat{k}$ only (b) $-\hat{k}$ only (c) $\frac{\hat{\imath}-\hat{\jmath}}{\sqrt{2}}$ (d) both (a) & (b)

Sol. (d)

SET B2 /5 If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a}| = 1, |\vec{b}| = 2$ and $\vec{a}\,.\,\vec{b} = \sqrt{3}$, then the angle between $2\vec{a}$ and $-\vec{b}$ is:

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{5\pi}{6}$ (d) $\frac{11\pi}{6}$

Sol. (c)

15/4. The value of $\hat{\imath}\,.\,(\hat{k} \times \hat{\jmath}) + \hat{\jmath}\,.\,(\hat{\imath} \times \hat{k}) + \hat{k}.\,(\hat{\imath} \times \hat{\jmath})$

(a) -3 (b) -2 (c) -1 (d) 0

Sol. (c)

16. The objective function $Z = ax + by$ of an LPP has maximum value 42 at $(4, 6)$ and minimum value 19 at $(3, 2)$. Which of the following is true?

(a) $a = 9, b = 1$ (b) $a = 5, b = 2$ (c) $a = 3, b = 5$ (d) $a = 5, b = 3$

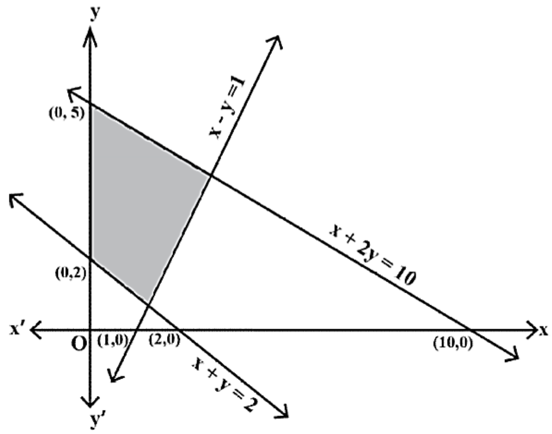
Sol. (c)

SET B2/3 The objective function $Z = ax + by$ of an LPP has maximum value 36 at $(4, 4)$ and minimum value 15 at $(3, 1)$. Which of the following is true?

(a) $a = 3, b = 5$ (b) $a = 3, b = 6$ (c) $a = 2, b = 7$ (d) $a = 5, b = 3$

Sol. (b)

17/2. The feasible region corresponding to the linear constraints of a Linear Programming Problem is given below.



Which of the following is not a constraint to the given Linear Programming Problem?

(a) $x + y \geq 2$ (b) $x + 2y \leq 10$ (c) $x - y \geq 1$ (d) $x - y \leq 1$

Sol. (c)

18/1. If A and B are independent events, then $P(\bar{A}/\bar{B}) =$

(a) $P(A/\bar{B})$ (b) $1 - P(A)$ (c) $1 - P(B)$ (d) $1 - P(\bar{A}/B)$

Sol. (b)

Assertion-Reasoning Based Question:

In the questions 19, 20 a statement of **Assertion (A)** is followed by a statement of **Reason (R).** Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true and R is not the correct explanation of A.
(c) A is true but R is false. (d) A is false but R is true.

19/20 **Assertion (A):** The value of $\cos\left[\frac{\pi}{2} + \sin^{-1}\left(-\frac{1}{2}\right)\right] = \frac{1}{2}$.

Reason (R): $\sin^{-1}(-x) = \sin^{-1}(x)$

Sol. (c)

20/19 **Assertion (A):** If $\left|\vec{a} \times \vec{b}\right|^2 + \left|\vec{a} \,.\, \vec{b}\right|^2 = 256$ and $\left|\vec{b}\right| = 8$, then $|\vec{a}| = 2$.

Reason (R): $\sin^2\theta + \cos^2\theta = 1$ and $\left|\vec{a} \times \vec{b}\right| = |\vec{a}|\left|\vec{b}\right| \sin\theta$ and $\left|\vec{a} \,.\, \vec{b}\right| = |\vec{a}|\left|\vec{b}\right| \cos\theta$

Sol. (a)

Section-B

(This section comprises 5 very short answer type-questions (VSA) of 2 marks each.)

21(a)/25(a) If $\cot^{-1}(3x + 5) > \frac{\pi}{4}$, then find the range of the values of x.

Sol. $\cot^{-1}(3x + 5) > \frac{\pi}{4} = \cot^{-1}(1)$ (0.5)

$\Rightarrow 3x + 5 < 1$ (as $\cot^{-1}(x)$ is strictly decreasing function in its domain) (0.5)

$\Rightarrow x < -\frac{4}{3} \Rightarrow \therefore x \in \left(-\infty, -\frac{4}{3}\right)$ (1)

OR

21(b)/25(b) Find the value of $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left[\sin\left(-\frac{\pi}{2}\right)\right]$

Sol. $-\frac{\pi}{6} + \frac{\pi}{3} + \tan^{-1}[-1]$ (1)

$\Rightarrow -\frac{\pi}{6} + \frac{\pi}{3} - \frac{\pi}{4} = -\frac{\pi}{12}$ (1)

22/24. If $xy = e^{x-y}$, then show that $\frac{dy}{dx} = \frac{y(x-1)}{x(y+1)}$

Sol. $xy = e^{x-y}$, taking log both side

$\Rightarrow \log(xy) = \log e^{x-y} \Rightarrow \log x + \log y = x - y$ (0.5)

Differentiate w.r.t $'x'$

$\Rightarrow \frac{1}{x} + \frac{1}{y}\frac{dy}{dx} = 1 - \frac{dy}{dx}$ (0.5)

$\Rightarrow \frac{dy}{dx}\left(1 + \frac{1}{y}\right) = 1 - \frac{1}{x}$

$\Rightarrow \frac{dy}{dx} = \frac{y(x-1)}{x(y+1)}$ (1)

23(a)/23(a) Find $\int e^x(\cos x - \sin x)\operatorname{cosec}^2 x\, dx$

Sol. $\int e^x(\cos x - \sin x)\operatorname{cosec}^2 x\, dx = \int e^x(\cot x \operatorname{cosec} x - \operatorname{cosec} x)dx$

$= \int e^x(\cot x \operatorname{cosec} x)dx - \int e^x \operatorname{cosec} x\, dx$ (0.5)

$= \int e^x(\cot x \operatorname{cosec} x)dx - (e^x \operatorname{cosec} x - \int -\operatorname{cosec} x \cot x \,.\, e^x dx + C)$ (1)

(using by parts in 2nd Integral)

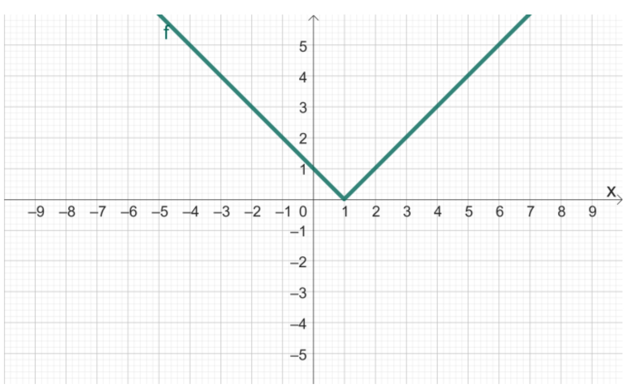
$= \int e^x(\cot x \operatorname{cosec} x)dx - e^x \operatorname{cosec} x - \int(\operatorname{cosec} x \cot x)e^x dx + C$

$= -e^x \operatorname{cosec} x + C$ (0.5)

OR

23(b)/23(b) Sketch the graph of $y = |x-1|$ and evaluate $\int_0^2 |x-1|\, dx$.

Sol.



(1 for fig)

$\int_0^2 |x-1|\, dx = \int_0^1 -(x-1)\, dx + \int_1^2 x - 1\, dx = \frac{1}{2} + \frac{1}{2} = 1$ (1)

24/22. If $f(x) = \begin{cases} 2x-3, & -3 \le x \le -2 \\ x+1, & -2 < x \le 0 \end{cases}$, Check the differentiability of $f(x)$ at $x = -2$.

Sol. $Lf'(-2) = \lim_{h\to 0}\frac{(f(-2-h) - f(-2))}{-h} = \lim_{h\to 0}\frac{(2(-2-h) - 3 - (-7))}{-h} = 2$ (1)

$Rf'(-2) = \lim_{h \to 0} \frac{(f(-2+h)-f(-2))}{h} = \lim_{h \to 0} \frac{((-2+h)+1-(-7))}{-h} = \lim_{h \to 0} \frac{6+h}{h}$,

Which does not exist, i.e., R.H.D does not exit

Therefore, the function is not differentiable at -2 (1)

Note: (1) If a student finds only RHD and concludes the result, full marks may be awarded.

(2) If a student proves that the function is discontinuous at -2 and hence not differentiable at -2, full marks may be awarded.

25/21. Express the vector $\vec{a} = 5\hat{\imath} - 2\hat{\jmath} + 5\hat{k}$ as the sum of two vectors such that one is parallel to the vector $\vec{b} = 3\hat{\imath} + \hat{k}$ and the other is perpendicular to $\vec{b}$.

Sol. Let $\vec{a} = \vec{x} + \vec{y} \Rightarrow \vec{y} = \vec{a} - \vec{x}$

Now, $\vec{x} \parallel \vec{b} \Rightarrow \vec{x} = t\vec{b} = 3t\hat{\imath} + t\hat{k}$ (0.5)

$\therefore \vec{y} = \vec{a} - \vec{x} = (5-3t)\hat{\imath} - 2\hat{\jmath} + (5-t)\hat{k}$

Also, $\vec{y} \perp \vec{b} \Rightarrow \vec{y} \, . \, \vec{b} = 0 \Rightarrow (5-3t).3 + (5-t) = 0 \Rightarrow t = 2$ (1)

$\therefore \vec{x} = 6\hat{\imath} + 2\hat{k}$ & $\vec{y} = -\hat{\imath} - 2\hat{\jmath} + 3\hat{k}$ (0.5)

Section-C

(This section comprises of short answer type-questions (SA) of 3 marks each.)

26(a)/31(a) If $x = a\sin^3\theta,\ y = b\cos^3\theta$, then find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{4}$.

Sol. We have, $y = b\cos^3\theta \Rightarrow \frac{dy}{d\theta} = -3b\cos^2\theta.\sin\theta$ (0.5)

Similarly, $x = a\sin^3\theta \Rightarrow \frac{dx}{d\theta} = 3a\sin^2\theta.\cos\theta$ (0.5)

$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = -\frac{b}{a}\cot\theta$

$\Rightarrow \frac{d^2y}{dx^2} = -\frac{b}{a}(-\mathrm{cosec}^2\theta).\frac{d\theta}{dx} = \frac{b}{a}\left(\mathrm{cosec}^2\theta \times \frac{1}{3a\sin^2\theta.\cos\theta}\right)$ (1)

$\Rightarrow$ At $\theta = \frac{\pi}{4}, \frac{d^2y}{dx^2} = \frac{4\sqrt{2}\,b}{3a^2}$ (1)

OR

26(b)/31(b) If $x^{13}y^7 = (x+y)^{20}$, then prove that $\frac{dy}{dx} = \frac{y}{x}$

Sol. Given, $x^{13}y^7 = (x+y)^{20}$

Taking logarithm both sides, we get

$\Rightarrow \log(x^{13}y^7) = \log((x+y)^{20}) \Rightarrow 13\log x + 7\log y = 20\log(x+y)$ (1)

Differentiating both sides w.r.t x, we get

$\frac{13}{x} + \frac{7}{y}\frac{dy}{dx} = \frac{20}{x+y}\left(1 + \frac{dy}{dx}\right) \Rightarrow \frac{13}{x} - \frac{20}{x+y} = \left(\frac{20}{x+y} - \frac{7}{y}\right)\frac{dy}{dx}$ (1)

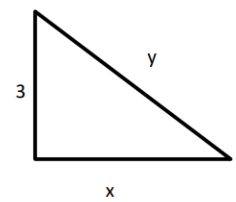
$\Rightarrow \frac{13x+13y-20x}{x(x+y)} = \left(\frac{20y-7x-7y}{y(x+y)}\right)\frac{dy}{dx} \Rightarrow \frac{13y-7x}{x(x+y)} = \left(\frac{13y-7x}{y(x+y)}\right)\frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx} = \frac{y}{x}$ hence proved. (1)

27/30. A kite is flying at a height of 3 metres and 5 metres string is out. If the kite is moving away horizontally at the rate of 200 cm/s, find the rate at which string is being released.

Sol. Let horizonal distance from person to kite = x m

Let length of the string = y m (0.5)



In triangle, using Pythagoras Theorem

$\Rightarrow x^2 + 3^2 = y^2 \ldots (1)$ (0.5)

Differentiating w.r.t $'t'$

$\Rightarrow 2x\frac{dx}{dt} = 2y\frac{dy}{dt}$ (1)

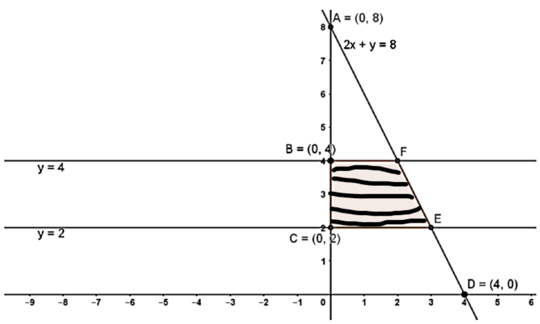
When $y = 5$, then $x = 4$ (from (1)) and $\frac{dx}{dt} = 200$ cm/s (given)

$\Rightarrow 2(4)(200) = 2(5)\frac{dy}{dt} \quad \Rightarrow \quad \frac{dy}{dt} = 160$ cm/s (1)

Hence, the rate at which string is being released is 160 cm/s

28(a)/29(a) Sketch the region bounded by the lines $2x + y = 8$, $y = 2$, $y = 4$ and the y-axis. Hence, obtain its area using integration.

Sol.



(1 for fig.)

Required Area = $\int_2^4 \frac{1}{2}(8 - y)\, dy$ (1)

$= \frac{1}{2}\left|8y - y^2\right|_2^4$ (0.5)

$= 5$ sq. units. (0.5)

OR

28(b)/29(b). Find the area of the following region using integration:

$\{(x, y): y^2 \leq 2x \text{ and } y \geq x - 4\}$

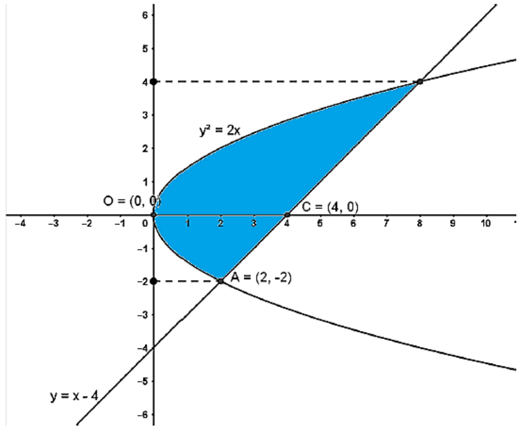
Sol. Solving $y^2 = 2x$ and $y = x - 4$, w gets $y = -2$ *or* 4 (0.5)

Required area = $\int_{-2}^{4}\left[(y + 4) - \frac{y^2}{2}\right] dy$ (1)

$= \left|\frac{y^2}{2} + 4y - \frac{1}{6}y^3\right|_{-2}^{4}$

(0.5)

= **18**



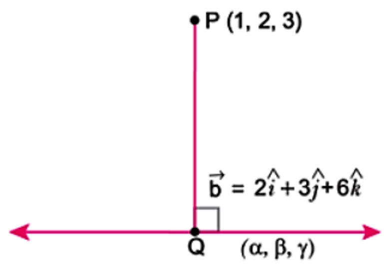
(1 for correct fig)

29(a)/28(a) A line passing through the point A with position vector $\vec{a} = 4\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$ is parallel to the vector $\vec{b} = 2\hat{\imath} + 3\hat{\jmath} + 6\hat{k}$. Find the length of perpendicular drawn on this line from a point P with position vector $\vec{r_1} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$

Sol. The equation of line passing through the point A and parallel to vector $\vec{b}$

$\Rightarrow \frac{x-4}{2} = \frac{y-2}{3} = \frac{z-2}{6} \ldots$(i) (0.5)

Let Q(α, β, γ) be foot of perpendicular drawn from the point P on The line (i)



Coordinate of P = (1, 2, 3) [as P.V of P is $\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$]

Since Q lies on line

$\therefore \frac{\alpha-4}{2} = \frac{\beta-2}{3} = \frac{\gamma-2}{6} = \lambda \Rightarrow \alpha = 2\lambda + 4, \beta = 3\lambda + 2, \gamma = 6\lambda + 2$ **(1)**

Now, $\overrightarrow{PQ} = (\alpha - 1)\hat{\imath} + (\beta - 2)\hat{\jmath} + (\gamma - 3)\hat{k}$

As, $\overrightarrow{PQ} \perp \vec{b} \Rightarrow \overrightarrow{PQ}.\vec{b} = 0 \Rightarrow 2\alpha + 3\beta + 6\gamma - 26 = 0$ **(0.5)**

Putting the values of α, β, γ; in above equation we get, $\lambda = 0$ **(0.5)**

Hence coordinate of Q = (4, 2, 2)

Length of PQ = $\sqrt{(4-1)^2 + (2-2)^2 + (2-3)^2} = \sqrt{10}$ units **(0.5)**

OR

29(b)/28(b) Find the vector and cartesian equation of the line passing through the point $(1, 2, -4)$ and perpendicular to two lines $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$

Sol. Let the D.R's of the required line be a, b, c. (0.5)

Since the required line is perpendicular to the given lines with D.R's $(3, -16, 7)$ and $(3, 8, -5)$, the dot product of their D.R's must be zero. (0.5)

$\therefore a(3) + b(-16) + c(7) = 0$ and $a(3) + b(8) + c(-5) = 0$

i.e., $3a - 16b + 7c = 0$ and $3a + 8b - 5c = 0$

$\Rightarrow \frac{a}{2} = \frac{b}{3} = \frac{c}{6}$, therefore D.R's are 2, 3 and 6 (1)

Hence cartesian equation of line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$ (0.5)

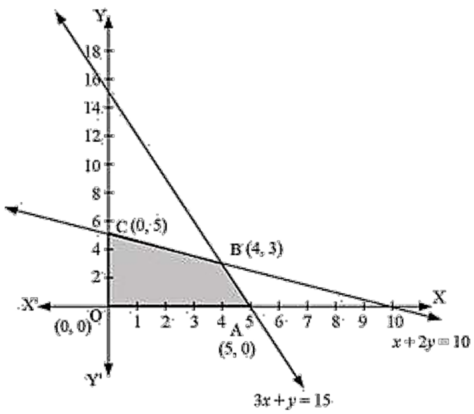
Vector Equation of line = $\vec{r} = \hat{\imath} + 2\hat{\jmath} - 4\hat{k} + \lambda(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k})$ (0.5)

30. Solve the following LPP graphically:

Maximize: $Z = 3x + 2y$

Subject to the constraints: $x + 2y \leq 10,\ 3x + y \leq 15,\quad x \geq 0,\ y \geq 0$

Sol.



(correct graph 1.5 marks)

Corner point	$Z = 3x + 2y$
$A(5,0)$	15
$B(4,3)$	18
$C(0,5)$	10

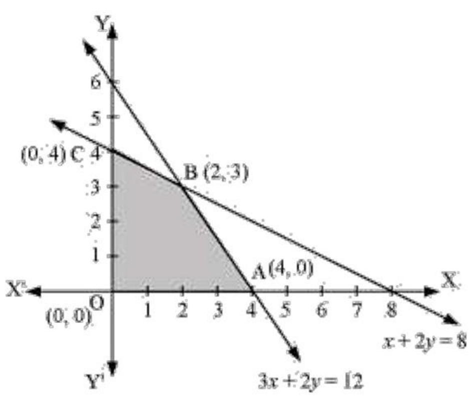
(Table 1 marks)

The maximum value of Z is 18, which is attained at $x = 4$, $y = 3$. (0.5)

SET B2/ 27. Solve the following LPP graphically:

Minimize: $Z = -3x + 4y$

Subject to the constraints: $x + 2y \leq 8,\ 3x + 2y \leq 12,\ x \geq 0,\ y \geq 0$



(correct graph 1.5 marks)

Corner point	$Z = -3x + 4y$
$O(0,0)$	0
$A(4,0)$	-12
$B(2,3)$	6
$C(0,4)$	16

(1 for table)

Min $Z = -12$, at $x = 4$ and $y = 0$ (0.5)

31/26. Consider the experiment of tossing a coin. If the coin shows head, toss it again; but if it shows a tail, then throw a die. Find the conditional probability of the event A: 'the die shows a number greater than 3' given that B: 'there is at least one tail'.

Sol. Let A:The die shows a number greater than 3 and B:There is at least one tail

$P(B \cap A) = P(T4, T5, T6) = \frac{3}{12} = \frac{1}{4}$ (1)

$P(B) = P(HT, T1, T2, T3, T4, T5, T6) = \frac{1}{4} + \frac{6}{12} = \frac{3}{4}$ (1)

$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{1}{3}$ (1)

Section-D

(This section comprises of long answer type-questions (LA) of 5 marks each.)

32. Determine the product $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and hence use it to solve the equations $x - y + z = 4; x - 2y - 2z = 9; 2x + y + 3z = 1$

Sol. Let $A = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$

Then $AB = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix} = 8I$ (1)

$AB = 8I \Rightarrow B^{-1} = \frac{1}{8}A$ (0.5)

Given system of equations: $x - y + z = 4; x - 2y - 2z = 9; 2x + y + 3z = 1$

In matrix form it can be written as $BX = C$ (0.5)

Where, $B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $and\ C = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$ (0.5)

$|B| = 8 \neq 0$, So, the system is consistent and have unique solution given by (0.5)

$\Rightarrow X = B^{-1}.C = \frac{1}{8}A.C = \frac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}.\begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}$ (1)

Hence, $x = 3, y = -2, z = -1$ (0.5)

SET B2/35. If $A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix}$, find A^{-1} and use it to solve the following system of Equations: $x - 2y = 10; 2x - y - z = 8; \ -2y + z = 7$

Sol. $|A| = 1 \neq 0$ hence A^{-1} exists. (1)

$Adj\ A = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix}$ (2)

Therefor, $A^{-1} = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix}$

Now, Given system of equations: $x - 2y = 10; 2x - y - z = 8; \ -2y + z = 7$

In matrix form it can be written as $AX = B$

Where, $A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $and\ B = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$ (0.5)

$\Rightarrow X = A^{-1}.B = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix} . \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix} = \begin{bmatrix} 0 \\ -5 \\ -3 \end{bmatrix}$ (1.5)

$\Rightarrow x = 0, y = -5, z = -3$

33(a)/34(a). Evaluate: $\int_0^{\frac{\pi}{4}} \frac{dx}{\cos^3 x\sqrt{2\sin 2x}}$

Sol. $\int_0^{\frac{\pi}{4}} \frac{dx}{\cos^3 x\sqrt{2\sin 2x}} = \int_0^{\frac{\pi}{4}} \frac{dx}{\cos^3 x\sqrt{2.2\sin x\cos x}} = \frac{1}{2}\int_0^{\frac{\pi}{4}} \frac{dx}{\cos^3 x\sqrt{\sin x.\cos x}}$ (1)

$\Rightarrow \frac{1}{2}\int_0^{\frac{\pi}{4}} \frac{\sec^4 x dx}{\sqrt{\tan x}} = \frac{1}{2}\int_0^{\frac{\pi}{4}} \frac{\sec^2 x.\sec^2 x\ dx}{\sqrt{\tan x}} = \frac{1}{2}\int_0^{\frac{\pi}{4}} \frac{\sec^2 x.(1+\tan^2 x)\ dx}{\sqrt{\tan x}}$ (2)

Let $\tan x = t$, then As $x = 0 \Rightarrow t = 0$ & $x = \frac{\pi}{4} \Rightarrow t = 1$

$\therefore I = \frac{1}{2}\int_0^1 \frac{1+t^2\ dx}{\sqrt{t}} = \frac{1}{2}\int_0^1 (t^{-\frac{1}{2}} + t^{\frac{3}{2}})\ dt = \frac{1}{2}\left[\frac{t^{\frac{1}{2}}}{\frac{1}{2}}\right]_0^1 + \frac{1}{2}\left[\frac{t^{\frac{5}{2}}}{\frac{5}{2}}\right]_0^1 = \frac{6}{5}$ (2)

OR

33(b)/34(b) Find $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x+\cos x}{\sqrt{\sin 2x}}\ dx$

Sol. $I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x+\cos x}{\sqrt{\sin 2x}}\ dx$

Let $\sin x - \cos x = t \Rightarrow (\sin x - \cos x)^2 = t^2$ (2)

$\Rightarrow 1 - \sin 2x = t^2 \Rightarrow \sin 2x = 1 - t^2$

Also, as $x \to \frac{\pi}{6}, t \to \frac{1-\sqrt{3}}{2}$ and $x \to \frac{\pi}{3}, t \to \frac{\sqrt{3}-1}{2}$ (0.5)

$I = \int_{\frac{1-\sqrt{3}}{2}}^{\frac{\sqrt{3}+1}{2}} \frac{dt}{\sqrt{1-t^2}} \Rightarrow [\sin^{-1} t]_{\frac{1-\sqrt{3}}{2}}^{\frac{\sqrt{3}-1}{2}} = 2\sin^{-1}\frac{\sqrt{3}-1}{2}$ (2.5)

34(a)/33(a) If a curve $y = f(x)$, passing through the point $(1, 2)$ is the solution of differential Equation: $2x^2 dy = (2xy + y^2)dx$, find the value of $f\left(\frac{1}{2}\right)$

Sol. We have $2x^2 dy = (2xy + y^2)dx$ ... ... (i)

$\Rightarrow \frac{dy}{dx} = \frac{2xy+y^2}{2x^2}$, given differential equation is a homogeneous differential equation

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ (0.5)

Using these values in (i)

$v + x\frac{dv}{dx} = \frac{x^2(2v+v^2)}{2x^2} \Rightarrow x\frac{dv}{dx} = \frac{(2v+v^2)}{2} - v \Rightarrow x\frac{dv}{dx} = \frac{v^2}{2} \Rightarrow 2\frac{dv}{v^2} = \frac{dx}{x}$ (1)

Integrating, we get

$\Rightarrow -\frac{2}{v} = \log|x| + C$

$\Rightarrow -\frac{2x}{y} = \log|x| + C \quad ... (ii)$ (1.5)

Since (ii) passes through $(1, 2)$

$\therefore -\frac{2}{2} = \log|1| + C \quad \Rightarrow \quad C = -1$ (0.5)

$\Rightarrow -\frac{2x}{y} = \log|x| - 1 \;\Rightarrow\; y = \frac{2x}{1-\log x}$ (0.5)

Also, $y\left(\frac{1}{2}\right) = \frac{1}{1+\log 2}$ (1)

OR

34(b)/33(b) If $\cos x \frac{dy}{dx} - y \sin x = 6x, 0 < x < \frac{\pi}{2}$ and $y\left(\frac{\pi}{3}\right) = 0$. Find the value of $y\left(\frac{\pi}{6}\right)$.

Sol. Given, $\cos x \frac{dy}{dx} - y \sin x = 6x$

$\Rightarrow \frac{dy}{dx} + \frac{(-y\sin x)}{\cos x} = \frac{6x}{\cos x}$ (0.5)

It's a linear differential equation

$I.F. = e^{\int -\frac{\sin x}{\cos x} dx} = e^{\log|\cos x|} = \cos x$ (1)

$\therefore$ Solution is $y \cos x = \int \frac{6x}{\cos x} \cos x \, dx$ (1)

$\Rightarrow y\cos x = \int 6x \, dx \Rightarrow y \cos x = 3x^2 + C$ (0.5)

As, $y\left(\frac{\pi}{3}\right) = 0 \qquad \Rightarrow \quad C = -\frac{\pi^3}{3}$ (0.5)

$\therefore y\cos x = 3x^2 - \frac{\pi^3}{3}$ (0.5)

$\therefore y\left(\frac{\pi}{6}\right) = \left(3\left(\frac{\pi}{6}\right)^2 - \frac{\pi^3}{3}\right)\sec\frac{\pi}{6} = -\frac{\pi^2}{2\sqrt{3}}$ (1)

35/32. Find the distance between the line $\frac{x}{2} = \frac{2y-6}{4} = \frac{1-z}{-1}$ and another line parallel to it passing through the point $(4, 0, -5)$.

Sol. Equation of the given line in standard form: $L_1 : \frac{x}{2} = \frac{y-3}{2} = \frac{z-1}{1}$ (0.5)

Equation of the line parallel to L_1 & passing through (4, 0, -5) is

$\frac{x-4}{2} = \frac{y}{2} = \frac{z+5}{1}$ (1)

Vector Equation of Lines are $L_1 = \vec{r} = 0\hat{\imath} + 3\hat{\jmath} + \hat{k} + \lambda(2\hat{\imath} + 2\hat{\jmath} + \hat{k})$

Vector Equation of Lines are $L_2 = \vec{r} = 4\hat{\imath} + 0\hat{\jmath} - 5\hat{k} + \lambda(2\hat{\imath} + 2\hat{\jmath} + \hat{k})$

Now, $\overrightarrow{a_2} - \overrightarrow{a_1} = (4\hat{\imath} + 0\hat{\jmath} - 5\hat{k}) - (0\hat{\imath} + 3\hat{\jmath} + \hat{k}) = 4\hat{\imath} - 3\hat{\jmath} - 6\hat{k}$ (0.5)

$\vec{b} = 2\hat{\imath} + 2\hat{\jmath} + \hat{k}; |\vec{b}| = \sqrt{4+4+1} = 3$ (0.5)

$(\overrightarrow{a_2} - \overrightarrow{a_1}) \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 4 & -3 & -6 \\ 2 & 2 & 1 \end{vmatrix} = 9\hat{\imath} - 16\hat{\jmath} + 14\hat{k}$ (1)

Distance between L_1 & L_2 $= \frac{|(\overrightarrow{a_2} - \overrightarrow{a_1}) \times \vec{b}|}{|\vec{b}|} = \frac{\sqrt{81+256+196}}{3} = \frac{\sqrt{533}}{3}$ units (1.5)

Section-E

(This section comprises of 3 case-based questions of 4 marks each with sub parts)

Q36/38. A class-room teacher is keen to assess the learning of her students the concept of "relations" taught to them. She writes the following five relations each defined on the set A = {1, 2, 3}:

$R_1 = \{(2, 3), (3, 2)\}$

$R_2 = \{(1, 2), (1, 3), (3, 2)\}$

$R_3 = \{(1, 2), (2, 1), (1, 1)\}$

$R_4 = \{(1, 1), (1, 2), (3, 3), (2, 2)\}$

$R_5 = \{(1, 1), (1, 2), (3, 3), (2, 2), (2, 1), (2, 3), (3, 2)\}$

The students are asked to answer the following questions about the above relations:

(i) Identify and justify the relation which is reflexive, transitive but not symmetric.

(ii) Identify and justify the relation which is reflexive and symmetric but not transitive.

(iii) (a) Identify and justify the relations which are symmetric but neither reflexive nor transitive.

OR

(iii) (b) What pairs should be added to the relation R_2 to make it an equivalence relation? Also find the number of equivalence relations possible on set A

Sol. (i) R_4 as $(1, 2) \in R_4$ but $(2,1) \notin R_4$ hence not symmetric and Reflexive and transitive by definitions (1)

(ii) R_5 as $(1, 2) \in R_5$ and $(2, 3) \in R_5$ but $(1,3) \notin R_5$ hence not transitive and reflexive, symmetric by definitions. (1)

(iii) (a) R_1 and R_3 as, (0.5)

$\Rightarrow (1,1) \notin R_1, (2,2) \notin R_1, (3,3) \notin R_1$ hence not reflexive and $(2, 3) \in R_1$ and $(3, 2) \in R_1$ but $(2,2) \notin R_1$ hence not transitive. (1)

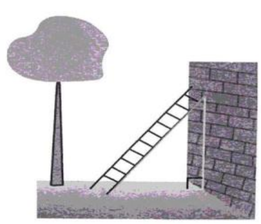
$\Rightarrow (2,2) \notin R_1, (3,3) \notin R_1$ hence not reflexive and $(2, 1) \in R_3$ and $(1, 2) \in R_3$ but $(2,2) \notin R_3$ hence not transitive. (0.5)

OR

(iii) (b) Required pairs to be added to make the relation R2 as an equivalence relation are: (1, 1), (2, 2), (3, 3), (2, 1), (3, 1) and (2, 3) (1)

Number of equivalence relations possible on set A = 5 (1)

Q37/37. A ladder of fixed length 'h' is to be placed along the wall such that it is free to move along the height of the wall.



Based upon the above information, answer the following questions:

(i) Express the distance (y) between the wall and foot of the ladder in terms of 'h' and height (x) on the wall at a certain instant. Also, write an expression in terms of h and x for the area (A) of the right triangle, as seen from the side by an observer.

(ii) Find the derivative of the area (A) with respect to the height on the wall (x), and find its critical point.

(iii) (a) Show that the area (A) of the right triangle is maximum at the critical point.

OR

(iii) (b) If the foot of the ladder whose length is 5 m, is being pulled towards the wall such that the rate of decrease of distance (y) is 2 m/s, then at what rate is the height on the wall (x) increasing, when the foot of the ladder is 3 m away from the wall?

Sol. (i) $y^2 = h^2 - x^2$ (0.5)

$A = \frac{1}{2}xy = \frac{1}{2}x\sqrt{h^2 - x^2}$ (0.5)

(ii) $\frac{dA}{dx} = \frac{1}{2}\sqrt{h^2 - x^2} + \frac{1}{2}x\frac{-x}{\sqrt{h^2-x^2}}$ (0.5)

$\frac{dA}{dx} = 0$ gives $x = \frac{h}{\sqrt{2}}$ (0.5)

(iii)(a) $A'' = \frac{1}{2}\frac{-4x\sqrt{h^2-x^2}-(h^2-2x^2)\frac{-x}{\sqrt{h^2-x^2}}}{h^2-x^2}$ is < 0 at $x = \frac{h}{\sqrt{2}}$ (1.5)

Hence A is maximum at critical point (0.5)

OR

(iii) (b) $y^2 = 25 - x^2$ hence $y = 3$ gives $x = 4$ (0.5)

$2y\frac{dy}{dt} = -2x\frac{dx}{dt}$ $\quad (\frac{dy}{dt} = -2\ m/s)$ (1)

$\frac{dx}{dt} = 1.5$ m/s (0.5)

38/36. A shop selling electronic items sells smartphones of only three reputed companies A, B and C because chances of their manufacturing a defective smartphone are only 5%, 4% and 2% respectively. In his inventory he has 25% smartphones from company A, 35% smartphones from company B and 40% smartphones from company C.

A person buys a smartphone from this shop.

(i) Find the probability that it was defective.

(ii) What is the probability that this defective smartphone was manufactured by company B?

Sol. Let E_1: Smartphone manufactured from company A

E_2: Smartphone manufactured from company B

E_3: Smartphone manufactured from company C

A = Smartphone is defective

$P(E_1) = 25\% = 0.25$; $P(E_2) = 35\% = 0.35$; $P(E_3) = 40\% = 0.4$

$P\left(\frac{A}{E_1}\right) = 5\% = 0.05$; $P\left(\frac{A}{E_2}\right) = 4\% = 0.04$; $P\left(\frac{A}{E_3}\right) = 2\% = 0.02$

(i) $P(defective) = P(A) = P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right)$

$= (0.25) \times 0.05 + (0.35) \times 0.04 + (0.40) \times 0.02$ (1.5)

$= 0.0345$ (0.5)

(ii) $P\left(\frac{E_2}{A}\right) = \frac{P(E_2)P\left(\frac{A}{E_2}\right)}{P(E_1)P\left(\frac{A}{E_1}\right)+P(E_2)P\left(\frac{A}{E_2}\right)+P(E_3)P\left(\frac{A}{E_3}\right)} = \frac{(0.35)\times 0.04}{(0.25)\times 0.05+(0.35)\times 0.04+(0.40)\times 0.02}$ (1.5)

$= \frac{28}{69}$ (0.5)