

CLASS XII
POST MID-TERM EXAMINATION (2025-26)
APPLIED MATHEMATICS (241)
SET-A (Marking key)

Max Marks:80

Time:3 HRS

General Instructions: Read the following instructions very carefully:

- i) This Question paper contains 38 questions. All questions are compulsory.
- ii) This Question paper is divided into five Sections - A, B, C, D and E.
- iii) In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
- iv) In Section B, Q. no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
- v) In Section C, Q. no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
- vi) In Section D, Q. no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
- vii) In Section E, Questions no. 36 to 38 are case study-based questions carrying 4 marks each.
- viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 2 questions in Section C, 2 questions in Section D and one sub-part each in 2 questions of Section E.

SECTION A

(Multiple Choice Questions) Each question carries 1 mark.

- 1. 20 litres of a mixture contains milk and water in the ratio 3:1. The amount of milk, in litres to be added to the mixture so as to have milk and water in the ratio 4:1 is
a) 7 b) 4 c) 5 d) 6 (c)
- 2. In a 1000 m race, A reaches the final point in 56 seconds and B reaches in 70 seconds. By how much distance does A beat B?
a) 100m b) 120m c) 150m d) 200m (d)
- 3. If $y = ae^{mx} + be^{-mx}$, then $\frac{d^2y}{dx^2} =$
a) m^2y b) my c) $-m^2y$ d) $-my$ (a)

4. The component of a time series attached to long term variations is termed as
 a) Seasonal b) Irregular c) Secular Trend d) Cyclic **(c)**
5. The present value of a perpetuity of Rs. 750 payable at the beginning of each year, if money is worth 5% pa, is
 a) Rs.15000 b) Rs.15750 c) Rs.14250 d) Rs.14750 **(b)**
6. If x is the least non-negative integer satisfying $218 \equiv x \pmod{7}$, then $(x^2 + 1)$ is equal to
 a) 1 b) 2 c) 5 d) 50 **(b)**
7. If $A = \begin{pmatrix} 4 & 1 \\ 3 & 2 \end{pmatrix}$ and $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, then $A^2 - 6A$ is equal to
 a) $3I$ b) $5I$ c) $-3I$ d) $-5I$ **(d)**
8. Area (in square units) bounded by $x - y + 2 = 0$, $x = 0$, $x = 2$ and $y = 0$ is:
 a) 6 b) 8 c) 5 d) 7 **(a)**
9. In testing the statistical hypothesis, which of the following statement is false?
 a) The critical region is the values of the test statistic for which we reject the null hypothesis.
 b) The level of significance is the probability of Type-I error.
 c) In testing $H_0: \mu = \mu_0$, $H_a: \mu \neq \mu_0$ the critical region is two sided.
 d) The p-value measures the probability that the null hypothesis is true. **(d)**
10. Two groups of students from different schools are tested for their average mathematics scores Group One has 15 students, and group 2 has 12 students what is the degree of freedom for this independent two-sample t test?
 a) 20 b) 22 c) 25 d) 27 **(c)**
11. If A is a matrix of order 3×2 , then the order of AA^T is
 a) 2×2 b) 3×3 c) 2×3 d) 3×2 **(b)**
12. The sum of the order and degree of the differential equation $\left(1 + \frac{dy}{dx}\right)^3 = \left(\frac{d^2y}{dx^2}\right)^2$ is:
 a) 5 b) 4 c) 3 d) 6 **(b)**
13. The value of k for which the matrix $A = \begin{pmatrix} 2 & 10 \\ 5k-2 & 15 \end{pmatrix}$ is singular matrix.
 a) $k = 1$ b) $k = -1$ c) $k = 2$ d) $k = -2$ **(a)**

14. If each EMI is used to pay interest and part of the principal, then loan is said to be
 a) Paid off b) Amortized c) Discounted d) Well settled **(b)**
15. The corner points of the feasible region for an LPP are (0,10), (5,5), (15,15) and (0,20). If the objective function is $Z = px + qy, p, q > 0$, then the condition on p and q , so that the maximum of Z occurs at (15,15) and (0,20) is:
 a) $p = q$ b) $p = 2q$ c) $q = 2p$ d) $q = 3p$ **(d)**
16. A newspaper printing machine costs Rs.4,80,000 and estimated scrap value of Rs.25000 at the end of its useful life of 10 years. What is its annual depreciation as per linear method?
 a) Rs.4,550 b) Rs.45,500 c) Rs.50,500 d) Rs.61,500 **(b)**
17. For a binomial variate X , if $n = 4$ and $P(X = 0) = \frac{16}{81}$, then $P(X = 4)$ is
 a) $\frac{1}{3}$ b) $\frac{1}{27}$ c) $\frac{1}{81}$ d) $\frac{1}{16}$ **(c)**
18. If $A = [a_{ij}]$ is a matrix of order 2×2 such that $|A| = -15$ and C_{ij} represents the cofactor of a_{ij} , then value of $a_{21}C_{21} + a_{22}C_{22}$ is:
 a) -15 b) 15 c) 0 d) 1 **(a)**

Questions number 19 and 20 are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R).

Select the correct answer from the options (a), (b), (c) and (d) as given below:

- a) Both Assertion(A) and Reason(R) are true and (R) is the correct explanation of Assertion.
 b) Both Assertion(A) and Reason(R) are true but (R) is not the correct explanation of Assertion.
 c) Assertion(A) is true but Reason(R) is false.
 d) Assertion(A) is false but Reason(R) is true.

19. Assertion (A): For non-singular matrix A , $(A^2)^{-1} = (A^{-1})^2$.

Reason (R): $(AB)^{-1} = B^{-1}A^{-1}$. **(a)**

20. Assertion (A): If X is a Poisson random variable with parameter 3, then mean of X is 3 and variance of X is also 3.

Reason (R): If X is a Poisson random variable with parameter λ then mean of X is λ and variance of X is λ^2 . **(c)**

SECTION-B

(This section comprises of very short answer type-questions (VSA) of 2 marks each.)

21. (A) Find the remainder when $(6^{20} + 98!)$ is divided by 7.

Sol. We know $6^2 \equiv 1(mod 7) \Rightarrow (6^2)^{10} \equiv 1^{10}(mod 7) \Rightarrow$ remainder is 1

And 98! Is divisible by 7, therefore no remainder.

Hence, remainder when $(6^{20} + 98!)$ is divided by 7 is 1.

(1+1) M

OR

21. (B) If x is a positive real number, then find the smallest value of $\frac{x^2+361}{x}$.

Sol. $\frac{x^2+361}{x} = x + \frac{361}{x} \geq 2\sqrt{x \times \frac{361}{x}} \geq 2 \times 19 = 38$. Hence, minimum value is 38. **(1+1) M**

22. Suppose that X has a Poisson distribution. If $P(X = 2) = \frac{2}{3}P(X = 1)$, evaluate $P(X = 0)$.

Sol. Given $P(X = 2) = \frac{2}{3}P(X = 1) \Rightarrow \frac{e^{-\lambda}\lambda^2}{2!} = \frac{2}{3} \frac{\lambda e^{-\lambda}}{1!} \Rightarrow \lambda = \frac{4}{3}$.

Hence, $P(X = 0) = e^{-\lambda} = e^{-\frac{4}{3}}$.

(1+1) M

23. The annual depreciation of a car is ₹30000. If the scrap value of the car after 15 years is ₹50000, find the original cost of the car using linear method.

Sol. Given annual depreciation = ₹30000, Scrap value of car = ₹

Useful life = 15 years. Let the original cost of the car be C , then using

annual depreciation = $\frac{\text{original cost} - \text{scrap value}}{\text{useful life}}$, we get $30000 = \frac{C - 50000}{15}$

$\Rightarrow C = 500000$. Original cost of the car was ₹500000.

(1+1) M

24. (A) There are 5% defective items in a large bulk of items. What is the probability that a sample of 10 items will not include more than 1 defective item?

Sol. Let E be the event of 'item is defective'. Here $p = P(E) = \frac{5}{100} = \frac{1}{20}$, $q = \frac{19}{20}$ and

$n = 10$. Required probability = $P(X \leq 1) = P(0) + P(1) = (q + 10p)q^9$

$= \left(\frac{19}{20} + \frac{10}{20}\right) \left(\frac{19}{20}\right)^9 = \left(\frac{29}{20}\right) \left(\frac{19}{20}\right)^9$

(1+1) M

OR

24. (B) The probability distribution of a random variable X is given by

X	0	1	2	3
P(X)	k	$\frac{k}{2}$	$\frac{k}{4}$	$\frac{k}{8}$

i) Determine the value of k .

ii) Find $P(X \leq 2)$.

Sol. We know $k + \frac{k}{2} + \frac{k}{4} + \frac{k}{8} = 1 \Rightarrow k = \frac{8}{15}$ and

$P(X \leq 2) = P(0) + P(1) + P(2) = \frac{7}{4}k = \frac{7}{4} \times \frac{8}{15} = \frac{14}{15}$.

(1+1) M

25. Diwakar invested ₹20,000 in a mutual fund in year 2010 the value of mutual fund increases to ₹32,000 in year 2015 calculate the compound annual growth rate of his investment.

Given $(1.6)^{\frac{1}{5}} = 1.098$.

Sol. Given beginning value of the investment: ₹20,000,

Final value of investment: ₹32,000.

$$\begin{aligned}\text{Number of years: 5. So, CAGR} &= \left[\left(\frac{F.V.}{P.V.} \right)^{\frac{1}{n}} - 1 \right] \times 100 = \left[\left(\frac{32000}{20000} \right)^{\frac{1}{5}} - 1 \right] \times 100 \\ &= \left[1.6^{\frac{1}{5}} - 1 \right] \times 100 = (1.098 - 1) \times 100 = 9.8\%. \quad (1+1) \text{ M}\end{aligned}$$

SECTION-C

(This section comprises of short answer type questions (SA) of 3 marks each.)

26. A motorboat can travel at 5 km/hr in still water. It travelled 90 km downstream in a river and then returned, taking all together 100 hours. Find the rate of flow of the river.

Sol. Speed of the boat in still water = 5 km/hr and let rate of flow of river = y km/hr

$\therefore$ Speed of boat in upstream = $(5 - y)$ km/hr and in downstream = $(5 + y)$ km/hr

$$\therefore \frac{90}{5+y} + \frac{90}{5-y} = 100 \Rightarrow 90(5 - y + 5 + y) = 100(25 - y^2)$$

$$\Rightarrow y^2 = 16 \Rightarrow y = 4 \text{ km/hr. Hence, rate of flow of river is 4 km/hr. } (1+1+1) \text{ M}$$

27. (A) If $y = \sqrt{3x + 2}$, prove that $y \frac{d^2y}{dx^2} + \left(\frac{dy}{dx} \right)^2 = 0$.

Sol. Given $y = \sqrt{3x + 2}$, On differentiating w.r.t. x , we get

$$\frac{dy}{dx} = \frac{1}{2}(3x + 2)^{-\frac{1}{2}} \cdot 3 = \frac{3}{2\sqrt{3x+2}} = \frac{3}{2y} \Rightarrow y \frac{dy}{dx} = \frac{3}{2} \text{ again differentiating w.r.t. } x, \text{ we get}$$

$$y \cdot \frac{d^2y}{dx^2} + \frac{dy}{dx} \cdot \frac{dy}{dx} = 0 \Rightarrow y \frac{d^2y}{dx^2} + \left(\frac{dy}{dx} \right)^2 = 0. \quad (1+1+1) \text{ M}$$

OR

27. (B) The volume of a spherical ball increases at the rate of $3 \text{ cm}^3/\text{sec}$. Find the rate of change of its surface area when its radius is 2 cm.

Sol. Let volume is $V = \frac{4}{3}\pi r^3 \dots (i)$ and $S = 4\pi r^2 \dots (ii)$ Differentiating w.r.t. x , we

$$\text{get } \frac{dV}{dt} = \frac{4}{3}\pi \cdot 3r^2 \frac{dr}{dt} = 4\pi r^2 \frac{dr}{dt}. \text{ But } \frac{dV}{dt} = 3 \text{ cm}^3/\text{sec}. \therefore 3 = 4\pi r^2 \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{3}{4\pi r^2}$$

$$\text{Now, } \frac{dS}{dt} = 4\pi \cdot 2r \frac{dr}{dt} = 8\pi r \cdot \frac{3}{4\pi r^2} \Rightarrow \frac{dS}{dt} = \frac{6}{r}. \text{ when } r = 2 \text{ cm, } \frac{dS}{dt} = \frac{6}{2} = \frac{3 \text{ cm}^2}{s}.$$

(1+1+1) M

28. (A) A machine costs a company Rs.525000 and its effective life is estimated to be 20 years. A sinking fund is created for replacing the machine at the end of its lifetime when its scrap realizes a sum of Rs.25000 only. Calculate what amount should be provided every year out of profits for the sinking fund if it accumulates an interest of 5% per annum.

(Use $(1.05)^{20} = 2.653$)

Sol. Cost of new machine: Rs. 525000. Scrap value of old machine: Rs. 25000.

Hence, the money required for new machine: $525000 - 25000 = \text{Rs. } 500000$.

Here, $A = \text{Rs. } 500000, r = 5\% \Rightarrow i = \frac{5}{100} = 0.05, n = 20, R = ?$

$$\text{Use } A = R \left[\frac{(1+i)^n - 1}{i} \right] \Rightarrow R = \frac{500000 \times 0.05}{(1.05)^{20} - 1} = \frac{25000}{2.653 - 1} = \frac{25000}{1.653} = 15124 (\text{approx.})$$

Hence, amount of each annual deposit is Rs.15124.

(1+1+1) M

OR

28. (B) A ₹2000, 8% bond is redeemable at the end of 10 years at Rs 105. Find the purchase price to yield 10% effective rate. (Use $(1.1)^{-10} = 0.3855$)

Sol. Face value of the bond $C = ₹2000$. As the bond is redeemable at ₹105, so redemption price of the bond is 105% of its face value.

Therefore, redemption value $C = 1.05 \times ₹2000 = ₹2100$. Nominal rate $i = 8\% = 0.08$.

So, $R = C \times i = 2000 \times 0.08 = ₹160$. No. of periods before redemption $n = 10$ and annual yield rate $i = 10\% = 0.1$. Therefore, purchase price V is:

$$V = R \left[\frac{1 - (1+i)^{-n}}{i} \right] + C(1+i)^{-n} = 160 \left[\frac{1 - (1.1)^{-10}}{0.1} \right] + 2100(1.1)^{-10}$$

$$= 160 \left[\frac{1 - 0.3855}{0.1} \right] + 2100(0.3855) = 160 \times 6.14 + 809.6 = 1792.$$

Hence, the present value of the bond is ₹1792.

(1+1+1) M

29. A cistern can be filled by an inlet pipe in 20 hours and can be emptied by an outlet pipe in 25 hours. Both the pipes are opened. After 10 hours, the outlet pipe is closed, find the total time taken to fill the tank.

Sol. Portion of tank filled by inlet pipe in 1 hr. = $\frac{1}{20}$ and

Portion of tank emptied in 1 hr. = $\frac{1}{25}$. Net work done by pipes in 1 hr = $\frac{1}{20} - \frac{1}{25} = \frac{1}{100}$.

In 10 hours, portion of tank filled = $10 \times \frac{1}{100} = \frac{1}{10}$. Now outlet pipe is closed and

remaining $\frac{9}{10}$ portion of tank is filled by inlet pipe only.

So, time taken by inlet pipe = $20 \times \frac{9}{10} = 18 \text{ hrs.}$ Hence, total time taken = $10 + 18 = 28 \text{ hrs.}$

(1+1+1) M

30. The mean and variance of a random sample of 64 observations were computed as 160 and 100 respectively.

i) Compute the 95% confidence limits for population mean.

ii) In view of 95% confidence interval in estimating population mean, if error should not exceed ± 1.4 , how many additional observations are required?

Sol. i) Given, $\alpha = 0.05 \Rightarrow 1 - \alpha = 0.95$ and $\bar{x} = 160$ & $\sigma = 10$

$$E = Z_{\alpha/2} \left(\frac{\sigma}{\sqrt{n}} \right) = Z_{0.025} \left(\frac{10}{\sqrt{64}} \right) = 1.96 \frac{10}{8} = 2.45$$

95% confidence level: $(\bar{x} - E, \bar{x} + E) = (160 - 2.45, 160 + 2.45) = (157.55, 162.45)$

ii) $Z_{\alpha/2} \left(\frac{\sigma}{\sqrt{n}} \right) = 1.4 \Rightarrow 1.96 \times \frac{10}{\sqrt{n}} = 1.4 \Rightarrow \sqrt{n} = \frac{19.6}{1.4} = 14$

$\Rightarrow n = 14 \times 14 = 196$. Hence, 132 more observations are required. **(2+1) M**

31. A company manufactures two types of product A and B. Each unit of A requires 3 grams of nickel and 1 gram of chromium, while each unit of B requires 1 gram of nickel and 2 grams of chromium. The firm can produce 9 grams of nickel and 8 grams of chromium. The profit is ₹40 on each of product A and ₹50 on each unit of type B. Formulate the Linear Programming Problem (LPP) to maximise the problem.

Sol. Let x and y be the number of products of types A and B respectively manufactured company. As the profit on each product of type A is ₹40 and on each product of type B is ₹50, so the total profit is $Z = 40x + 50y$ (in ₹).

Hence, the problem can be formulated as an LPP as follows:

Maximize $Z = 40x + 50y$ subject to the constraints

$3x + 1y \leq 9$ (nickle constraint) ; $1x + 2y \leq 8$ (chromium constraint)

$x \geq 0, y \geq 0$ (number of products cannot be negative) **(1+1+1) M**

SECTION-D

(This section comprises of long answer type questions (LA) of 5 marks each.)

32. (A) If $A = \begin{pmatrix} 0 & 1 & 3 \\ 1 & 2 & a \\ 2 & 3 & 1 \end{pmatrix}$ and $A^{-1} = \frac{1}{2} \begin{pmatrix} 1 & -8 & 5 \\ -1 & 6 & -3 \\ 1 & 2b & 1 \end{pmatrix}$, find the values of a and b . Hence,

solve the given system of equations by matrix method: $y + 3z = 5, x + 2y + z = 8,$

$2x + 3y + z = 13$.

Sol. We know, $A^{-1}A = I \Rightarrow \frac{1}{2} \begin{pmatrix} 1 & -8 & 5 \\ -1 & 6 & -3 \\ 1 & 2b & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 3 \\ 1 & 2 & a \\ 2 & 3 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

$\Rightarrow \begin{pmatrix} 1 & 0 & 4-4a \\ 0 & 1 & -3+3a \\ b+1 & 2b+2 & 2+ab \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \Rightarrow a = 1 \text{ \& } b = -1 \text{ satisfy } 2 + ab = 1$.

Hence, $a = 1, b = -1$.

Given system can be written as $\begin{pmatrix} 0 & 1 & 3 \\ 1 & 2 & 1 \\ 2 & 3 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ 8 \\ 13 \end{pmatrix}$, where $X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ and $B = \begin{pmatrix} 5 \\ 8 \\ 13 \end{pmatrix}$.

Now, $X = A^{-1}B = \frac{1}{2} \begin{pmatrix} 1 & -8 & 5 \\ -1 & 6 & -3 \\ 1 & -2 & 1 \end{pmatrix} \begin{pmatrix} 5 \\ 8 \\ 13 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 5-64+65 \\ -5+48-39 \\ 5-16+13 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$

Hence, $x = 3, y = 2, z = 1$.

(2.5+2.5) M

OR

32. (B) A toy rocket is fired, from a platform, vertically into the air, its height above the ground after t seconds is given by $s(t) = at^2 + bt + c$, where $a, b, c \in R$; $a \neq 0$ and $s(t)$ is measured in metres. After 10 second, the rocket is 16 m above the ground; after 20 seconds, 22 m; after 30 seconds, 25 m.

(i) Write down a system of three linear equations in terms of a, b , and c .

(ii) Hence find the values of a, b and c , using Cramer's Rule.

Sol. Given $s(t) = at^2 + bt + c$; $t \geq 0$. Here, (10,16), (20,22), (30,25) lie on the curve of

$s(t)$. Therefore $100a + 10b + c = 16, 400a + 20b + c = 22, 900a + 30b + c = 25$.

Given system can be written as: $\begin{pmatrix} 100 & 10 & 1 \\ 400 & 20 & 1 \\ 900 & 30 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 16 \\ 22 \\ 25 \end{pmatrix}$.

$$\text{Now, } D = \begin{vmatrix} 100 & 10 & 1 \\ 400 & 20 & 1 \\ 900 & 30 & 1 \end{vmatrix} = -2000 \neq 0, D_x = \begin{vmatrix} 16 & 10 & 1 \\ 22 & 20 & 1 \\ 25 & 30 & 1 \end{vmatrix} = 30$$

$$D_y = \begin{vmatrix} 100 & 16 & 1 \\ 400 & 22 & 1 \\ 900 & 25 & 1 \end{vmatrix} = -2100, D_z = \begin{vmatrix} 100 & 10 & 16 \\ 400 & 20 & 22 \\ 900 & 30 & 25 \end{vmatrix} = -14000.$$

$$\text{Hence, } x = -\frac{3}{200}, y = \frac{21}{20}, c = 7.$$

(1.5+2+1.5) M

33. The cost function of a product is given by $C(x) = \frac{x^3}{3} - 45x^2 - 900x + 36$, where x is the number of units produced. In order to minimise the marginal cost, how many units of the product must be produced?

Sol. Given the cost function $C(x) = \frac{x^3}{3} - 45x^2 - 900x + 36$, Marginal cost = $MC = \frac{dC(x)}{dx}$.

$\Rightarrow MC = x^2 - 90x - 900$. To find values of x so that MC is minimum, put $\frac{dMC}{dx} = 0$ and

show that $\frac{d^2}{dx^2} MC > 0$. Here, $\frac{d}{dx} MC = 2x - 90$ and $\frac{d^2}{dx^2} MC = 2$. Also, $\frac{dMC}{dx} = 0$ at $x = 45$

and at $x = 45$, $\frac{d^2}{dx^2} MC = 2 > 0$. Hence, MC is minimum at $x = 45$. **(1+2+2) M**

34. (A) The production of a soft drink company in thousands of liters during each month of a year is as follows:

Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1.2	0.8	1.4	1.6	1.8	2.4	2.6	3.0	3.6	2.8	1.9	3.4

Calculate the five monthly moving averages and show these moving averages on a graph paper.

Solution.

Month	Monthly production	5-monthly moving total	5-month moving average
Jan	1.2		
Feb	0.8		
Mar	1.4	6.8	1.36
Apr	1.6	8.0	1.6
May	1.8	9.8	1.96
June	2.4	11.4	2.28
July	2.6	13.4	2.68
Aug	3.0	14.4	2.88
Sept	3.6	13.9	2.78
Oct	2.8	14.7	2.94

Nov	1.9		
Dec	3.4		

Graph **(3+2) M**

OR

34. (B) Fit a straight-line trend by the method of least square for the following data. Also, tabulate the trend values.

Year	2004	2005	2006	2007	2008	2009	2010
Profit (₹ '000)	114	130	126	144	138	156	164

Solution. Here, $n = 7$ (odd). So, middle year i.e., 2007 is taken as origin.

Year t	Profit y	x $= t_i - 2007$	x^2	xy	Trend values $y_t = a + bx$
2004	114	-3	9	-342	115.98
2005	130	-2	4	-260	123.58
2006	126	-1	1	-126	131.22
2007	144	0	0	0	138.86
2008	138	1	1	138	146.50
2009	156	2	4	312	154.14
2010	164	3	9	492	161.78
	$\Sigma y = 972$	$\Sigma x = 0$	$\Sigma x^2 = 28$	$\Sigma xy = 214$	

Now, $a = \frac{\Sigma y}{n} = \frac{972}{7} \Rightarrow a = 138.86$ and $b = \frac{\Sigma xy}{\Sigma x^2} = \frac{214}{28} \Rightarrow b = 7.64$

So, the required equation of the straight-line trend is: $y_t = 138.86 + 7.64x$. **(3+2) M**

35. Surjeet purchased a new house, costing Rs.40,00,000 and made a certain amount of down payment so that he can pay the balance by taking a home loan from XYZ Bank. If his equated monthly instalment is Rs.30,000 at 9% interest compounded monthly (reducing balance method) and payable for 25 years, then what is the initial down payment made by him? (Use $(1.0075)^{-300} = 0.1062$)

Sol. Given cost of new house: Rs.40,00,000. Let down payment be: Rs. x and

EMI: Rs.30,000 and $i = \frac{9}{1200} = 0.0075$, $n = 12 \times 25 = 300$. So, $P = \text{Rs. } (4000000 - x)$

Now, $EMI = P \times \frac{i}{1-(1+i)^{-n}} \Rightarrow 30000 = (4000000 - x) \times \frac{0.0075}{1-(1.0075)^{-300}}$

$\Rightarrow 30000 \times \frac{[1-(1.0075)^{-300}]}{0.0075} = 4000000 - x \Rightarrow 30000 \times \left[\frac{1-0.1062}{0.0075} \right] - 4000000 = -x$

$\Rightarrow x = 4000000 - 30000 \times 119.1733 = 4000000 - 3575200 = 4,24,800$.

Hence, initial down payment be Rs.4,24,800.

(1+2+2) M

SECTION- E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with sub parts. The first two case study questions have three sub parts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two sub parts of 2 marks each)

36. A game a man wins ₹8 for getting a number greater than 3 and loss ₹3 otherwise, when a fair die is thrown. The men decided to throw a die 4 times but to quit as and when he gets a number greater than 3.

Based on the above information answer the following questions:

- If X denotes the amount which the man wins or loses, then find all the possible values of X .
- Find the probability distribution of X .
- (a) Find the expected value of X .

OR

- (b) Find the variance of X .

Sol. i) Possible values: 8, 5, 2, -1, -12.

ii)

X	8	5	2	-1	-12
$P(X)$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{16}$

iii) (a) Mean = $\frac{75}{16}$

OR (b) Variance = $\frac{6615}{256}$

37. Analyzing the Grain Market for Rice

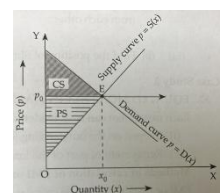
In the grain market for raise the relationship between price and quantity demanded can be modelled using a linear demand function suppose the following information is available from market data:

- At the price of ₹30 per kg the quantity demanded is 500 tons.
- At the price of ₹40 per kg in quantity demanded is 200 tons.
- The supply function is $p_s = -10 + \frac{x}{10}$.

Based on the above information, answer the following:

- Formulate the linear demand function based on the given data.
- Determine the equilibrium price and quantity.
- (a) Using integration, calculate the consumer surplus at the equilibrium price.

OR



iii) (b) Using integration, calculate the producer surplus at the equilibrium price.

Sol. i) Let demand function be $p_d = a + bx$. Using points (500,30) and (200,40),

$$\text{we get } p_d = \frac{140}{3} - \frac{1}{30}x.$$

ii) For equilibrium, $\frac{140}{3} - \frac{1}{30}x = -10 + \frac{x}{10}$, On solving $x = 425$ put in supply

equilibrium price: $p = -10 + \frac{425}{10} = 32.5$, equilibrium quantity = 425.

$$\begin{aligned} \text{iii) (a) } CS &= \int_0^{425} \left(\frac{140}{3} - \frac{1}{30}x - 32.5 \right) dx = \int_0^{425} \left(\frac{42.5}{3} - \frac{1}{30}x \right) dx = \left[\frac{42.5}{3}x - \frac{x^2}{60} \right]_0^{425} \\ &= 3010.41 \end{aligned}$$

OR

$$\begin{aligned} \text{iii) (b) } PS &= 425 \times 32.5 - \int_0^{425} \left(-10 + \frac{x}{10} \right) dx = 13812.5 - \left[-10x + \frac{x^2}{20} \right]_0^{425} \\ &= 13812.5 - \left(-4250 + \frac{425^2}{20} \right) = 9031.25. \end{aligned}$$

38. A furniture trader deals in only two items - chairs and tables. He has ₹50000 to invest and space to store at most 35 items. A chair costs him ₹1000 and a table costs him ₹2000. The trader earns a profit of ₹150 and ₹250 on a chair and a table, respectively. Trader wants to maximise the profit.

Based on the above information, answer the following:

i) Formulate the objective function and the constraints of the above Linear Programming Problem.

ii) Find the number of chairs and tables so that trader realizes maximum profit.

Sol. i) Let x be the number of chairs and y be the number of tables that the dealer buys and sells. Maximize $P = 150x + 250y$ subject to the constraints

$$x + y \leq 35 \quad (\text{storage constraint}); 1000x + 2000y \leq 50000$$

$$\text{ie } x + 2y \leq 50 \quad (\text{investment constraint}) \text{ and } x \geq 0, y \geq 0 \quad (\text{non-negativity})$$

ii) The feasible region is bounded with corner points: (0,0), (35,0), (20,15) and (0,25).
 P is maximum at (20,15) and maximum value of $P = 6750$.